| Mathematics Project Competition (2025/26)<br>數學專題習作比賽 (2025/26)<br>Information Sheet 資料頁 | | | | |
|---|---|---|---|---|
| Category<br>參賽組別 | ☐* A 組：初中習作　(Category A: Junior secondary project)<br>☑* B 組：中一小型習作 (Category B: S1 mini-project) | | | |
| Title of Project<br>專題習作題目 | The Math of Winning Uno: A Statistical Investigation | | | |
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| 學校名稱 | 拔萃女書院 | | | |
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* Please "✓" whichever is appropriate. 請在適用部分加上"✓"

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# The Math of Winning Uno: A Statistical Investigation

Abstract

Uno, as a popular card game, focuses not only on randomly drawing cards but also involves a more complex distribution of cards and strategy. Among players, the strategy "keeping the function cards with high value to execute defensive counter-attack" is a common strategy. To determine whether this strategy involves mathematical concepts, we utilized the AI Gemini to design a Python program for this experiment. Using Monte Carlo Simulation and simulating the game 10,000 times. This program compared the two strategies, which are "Aggressive Strategy" and "Defensive Strategy". According to the data, even though Defensive Strategy has a defence mechanism, Aggressive Strategy still has the advantage of winning Defensive Strategy by 59.79% with a higher overall average score. This essay will get into the topics "Expected Risk" and "Payoff Matrix", using mathematical logic to find the reason why the data is formed, proving that the Aggressive Strategy is the best mathematical way to win.

## 1. Introduction

### 1.1 Research Background

In this Game with Imperfect Information, the player does not know what cards does the opponent have, so the strategy mainly relies on probability, guess and intuition. In the Uno game, the most valued cards are Function Cards (Skip, Reverse and Draw Two) and Wild Cards (Wild Draw Four and Wild). Most players think that keeping the most valuable cards till the end can be a strong defensive strategy or a fatal attack on those who are about to win. This is called a "hoarding" logical thinking, creating the starting point of this research. By using the simulator, we hope to use objective mathematical concepts to verify whether this strategy has a mathematical misconception.

### 1.2 Research Objectives

The study was designed to investigate the below questions:

1. Does Defensive Strategy actually have an advantage compared to the Aggressive Strategy in the mathematical concepts?
2. Will the Defensive Strategy bring more loss than expected?
3. Can the Aggressive Strategy provide a higher probability to win?

This research is based on the mechanism for the Defensive Strategy (the most common) is to keep the function cards to counter-attack. This should be able to suppress the Aggressive Strategy, which will lead to victory for the player with the Defensive Strategy.

## 4. Results & Discussion

### 4.1 Simulation Results

Through 10000 simulations grounded in the Law of Large Numbers, we obtained the following data, which corroborate our deductions in Part 3.

| Strategy | Win Rate Performance | Total Points Scored | Average Points per Win |
|---|---|---|---|
| Aggressive Strategy (A1, A2) | 59.79% (5,979 Wins) | 594,082 Points | 99.36 Points/Win |
| Defensive Strategy (D1, D2) | 40.21% (4,021 Wins) | 352,757 Points | 87.73 Points/Win |

### 4.2 Net Expected Payoff

Furthermore, based on these statistical findings, we refine our mathematical model, ensuring it accurately reflects the gameplay dynamics by calculating Net Expected Payoff (*NEP*).

In the traditional games, payoffs are normally measured in “number of cards”. However, in UNO’s scoring system, the risk is usually calculated in “points”.

Based on this, we defined a formula to accurately measure the value of each card play:

$$\boldsymbol{NEP = S + O \times 4.74 - V \times P(L)}$$

*S*: self-interest, *O*: offence, *E*: hidden penalty expected value, *P*(*L*): Losing probability

## 4.3 Conclusion

When faced with the same decision, the aggressive strategy would have an expected profit of up to +68.96 points; while for the defensive strategy, in order to "preserve strength", their net expected profit would be 25.16 points.

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There is an "expected value gap" of **94.12 points** between the two strategies, which **match average points per win**. This model eliminates the randomness of player luck and is purely based on the law of large numbers and the logic of the game.

This data becomes the evidence that the defensive players try to exchange a hidden risk ( 50) for just a small progress (+4.74), which is a mathematically disadvantageous strategy. Therefore, in a game like UNO with random termination features, "attacking" is a better solution with a mathematical advantage.

## 5. Reflection & Further Development

After going through 10,000 experiments, we finally reached a conclusion, with the aggressive strategy winning approximately 59.79% (5,979 times) and its average point per game being 99.36 points, with a grand total of 594082 points and the defensive strategy winning about 40.21% of the time (4021 times), with an average of 87.73 points per game and a total of 352757 points.

Throughout the whole project, we learnt a lot about Uno and throughout our own trials of playing UNO together, we found that our own results varied slightly from the ones we reached from our experiments, as the aggressive strategy won mostly in our experience.

Before doing this project, we thought that the defensive strategy would be most likely to win as it saved up its cards to unleash on the opponent at last, while the aggressive strategy was less likely due to it using up its Draw cards quickly and we thought it would run out of function cards.

After finishing this project, we understood that although the defensive strategy seemed quite fool-proof and effective as it saved up all its attacking cards for the end, in reality it actually takes a larger risk since the other player may easily win before using up all its attacking cards like draw cards and they have a significantly higher penalty, which causes a larger loss.

Another thought about the defensive strategy after this experiment was that it would also miss important chances which originally had it chosen to attack, it would have had a really strong advantage over the opponent, but the defensive strategy would choose to wait and see if there were any better opportunities, but usually there would not be enough in the end.

On the other hand, the aggressive strategy went with the flow more as it always took opportunities to attack immediately which gained it an advantage over the defensive strategy. As to our previous thought about the aggressive strategy running out of offensive cards would give them a disadvantage over the defensive strategy, that would not be true as the only cards they have left would be number cards and at such a stage there would likely be very few number cards left, thus they would very possibly win quickly.

If we could do this project again, a detail we would change and reinvestigate would be in the Payoff Matrix for the Draw Two and Draw Four cards, as a scenario that could happen would be all the other players also playing a draw card after you initiated playing a draw card which would lead to you drawing a large number of cards. This would change the rate of disturbing the opponent as in the end it could quite possibly cause you a loss. We could reinvestigate this detail and research the possibilities of this specific scenario happening and in which strategy would it be more likely to happen, as we are guessing aggressive since the defensive strategy would not initiate a draw card as likely as the aggressive strategy.

## 6. Reference


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