# Arrangement for Introverts

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## 1 – Introduction

As a lonely introvert, it is important that I should always keep distant from strangers while I'm in public, to avoid potential awkwardness.

What if, everyone in public is an introvert, and they **all** want to keep distant from others? Where would they choose to stand, to keep themselves far from other people?

And this is the purpose of our project. If we have $n$ introverts together in the same room, we want to investigate about how we can arrange their positions, to maximize the *minimum pairwise distance*.

(The *minimum pairwise distance* is **minimum** of the distances between **all pairs of people**. For instance, given 3 people $P_1$, $P_2$ and $P_3$. If $P_1P_2 = 0.4$, $P_1P_3 = 0.3$, $P_2P_3 = 0.5$, the *minimum pairwise distance* is 0.3 in this case.)

In this project, we **denote $d$ as the *minimum pairwise distance***.

The room could have a lot of different shapes (e.g. a square, a circle). And it is a unique story of every different shape. Therefore, in this project, we will focus on 3 cases: the room is a straight line, a square, or a circle.

**Remarks:** For a certain number of cases, we failed to prove the optimality of our arrangement, although some of them are "obvious".

Therefore, we are just making a "guess" or "conjecture" for some of the cases and dedicate at improving the solution.

## 6 – Conclusion and Future Investigations

To conclude, it is easy to solve the problem for small values of $n$, but unexpectedly hard to solve it for large values of $n$.

In the approach using several layers of circle above, there's a critical problem I haven't mentioned yet.

Note that when we use 2 layers, we must distribute the points on the smaller and bigger circles in a ratio of 1:2.

(In order to "*balance*" the distance between each pair of adjacent points on the circumference on each circle)

However, only multiples of 3 can be divided into two integral parts with ratio 1:2.

This problem became more serious when the number of layers increases, as it is much harder for a number to be divisible by 1+2+3, 1+2+3+4, 1+2+3+4+5, etc.

For our calculations, we just substitute the values of $n$ and $k$ to get the answer. However, the arrangement couldn't be implemented geometrically.

For instance, when $n = 31$, we are trying to arrange them on 3 layers.

However, in this case, we should put $\frac{31}{6}$ points on the smallest circle, $\frac{31}{3}$ points on the second biggest circle, and $\frac{31}{2}$ points on the biggest one.

This is clearly unachievable.

In our calculations, we just substitute $n = 31$ and $k = 3$, and get the distance between 5.16666666… points, which is **ridiculous**.

As solving this problem may involve number theory (divisibility) and is beyond our current scope, we can't solve it now, causing most of the results to be inaccurate.

We could solve this problem in future investigations.

Moreover, the optimality of many "obvious" cases like $n = 3$ in a square and $n \geq 5$ in a circle have not yet been proved and can be proved in further investigations.

In fact, we realized that these cases are in fact extremely difficult to solve, after we asked out o3-mini for the solution of $n = 7$ on a square.

**Final Answer**

For $n = 7$, it is now widely accepted (based on extensive numerical work) that the optimal placement of 7 points in the unit square has minimal pairwise distance

$$d^* \approx 0.632455532.$$

A (conjecturally optimal) configuration achieving this value is (approximately)

- $P_1 \approx (0.000000000,\ 0.265201000)$,
- $P_2 \approx (0.265201000,\ 1.000000000)$,
- $P_3 \approx (0.734799000,\ 1.000000000)$,
- $P_4 \approx (1.000000000,\ 0.734799000)$,
- $P_5 \approx (1.000000000,\ 0.265201000)$,
- $P_6 \approx (0.734799000,\ 0.000000000)$,
- $P_7 \approx (0.265201000,\ 0.000000000)$.

No arrangement of 7 points inside the unit square can yield a minimum distance larger than approximately 0.632455532.

Although this solution is not correct (as $P_4P_5 = 0.376$, 3 s.f.), but we can see that solving this problem and proving optimality for large values of $n$ is so hard, as human beings don't possess the ability to solve it yet.

There’s good news though. The technology of quantum computers has been rising rapidly over recent years. With the aid of its superb computing powers, the problem may be solved by brute force.

However, it’s beyond our current scope.