Mathematics Project Competition (2024/25) 數學專題習作比賽 (2024/25) Information Sheet 資料頁				
Category 參賽組別	☐* A 組：初中習作 (Category A: Junior secondary project) ☑* B 組：中一小型習作 (Category B: S1 mini-project)			
Title of Project 專題習作題目	General Solution of an IQ Question			
Name of School	Sha Tin Government Secondary School			
學校名稱	沙田官立中學			
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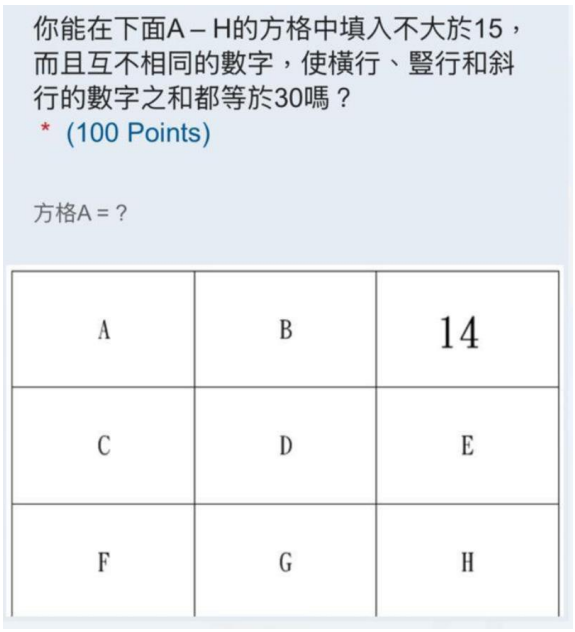
Introduction

There was an IQ question published by St. Paul's College Primary School, asking the following statement:

"Could you fill in a unique number that are not greater than 15 for each of the following letter, so that the sum of each row/column/diagonal are equal to 30?"



Although the question itself is not challenging, it is worth thinking if the question can be generalized. For instance, we may consider the following research questions:

- ***Are there a general solution to the problem?***
- ***Is the problem still solvable if the position of the given number is different?***
- ***What are the possible restrictions for the given numbers and the required sum?***

In this project, we hope to present the exploration of a general solution to this engaging IQ question, leveraging fundamental concepts from secondary school mathematics. Specifically, the question includes application of the mathematical topic "Solving Equations" (introduced in S.1), "Simultaneous Equations in Two Unknowns" (introduced in S.2), and "Gaussian Elimination" (introduced in M2) to devise a systematic and effective approach to this problem. By blending these techniques, our goal is to not only provide a practical solution but also demonstrate the connection of how these methods address complex mathematical challenges. This project showcases how foundational tools can be wielded creatively, offering both clarity and insight into the problem-solving process.

Conclusion

To conclude, the above IQ question can be generalized despite of the initial position of the given number and the required sum.

The question can be classified into three cases, namely the "Corner Entries Problem", "Middle-side Entries Problem" and the "Centre Entries Problem". The general solutions of them are $\begin{pmatrix} \frac{n}{3}-C+k & \frac{2n}{3}+C-2k & k \\ C & \frac{n}{3} & \frac{2n}{3}-C \\ \frac{2n}{3}-k & 2k-C & \frac{n}{3}+C-k \end{pmatrix}$, $\begin{pmatrix} A & k & n-k-A \\ \frac{4n}{3}-k-2A & \frac{n}{3} & \frac{-2n}{3}+k+2A \\ \frac{-n}{3}+k+A & \frac{2n}{3}-k & \frac{2n}{3}-A \end{pmatrix}$ and $\begin{pmatrix} \frac{2n}{3}-t & \frac{2n}{3}-s & \frac{-n}{3}+s+t \\ \frac{-2n}{3}+s+2t & \frac{n}{3} & \frac{4n}{3}-s-2t \\ n-s-t & s & t \end{pmatrix}$ respectively, where A, C, s and t are variables that we may define freely, and the domain of them are integers between $\left\lceil \frac{n}{6} \right\rceil$ and $\left\lfloor \frac{n}{2} \right\rfloor$.

However, there are several constraints for the general solution. For instance, all three general solutions contain the term $\frac{2n}{3}$ and $\frac{n}{3}$, therefore, the required sum must be a multiple of 3 in order to have an integer in the centre entry. Secondly, as the centre entry is always $\frac{n}{3}$, that means the entry is a constant, which makes the "Centre Entry Problem" no different to the "Middle-side Entries Problem".

Lastly, due to the limitation of the scale of this project, a more precise and narrower domain of the entries is yet to be discussed. We suggest later study to set up an inequality for every single entry of the solutions (e.g., $\frac{2n}{3}+C-2k \le \left\lfloor \frac{n}{2} \right\rfloor$), so that a system of inequalities can be form and hopefully a more precise constraint for the variables can be found by solving the system.

Reference

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