Mathematics Project Competition (2023/24) 數學專題習作比賽 (2023/24) Information Sheet 資料頁				
Category 參賽組別	☑* A 組：初中習作　(Category A: Junior secondary project) ☐* B 組：中一小型習作 (Category B: S1 mini-project)			
Title of Project 專題習作題目	Incenters and Inspheres of Tetrahedrons			
Name of School	Wah Yan College, Kowloon			
學校名稱	華仁書院 (九龍)			
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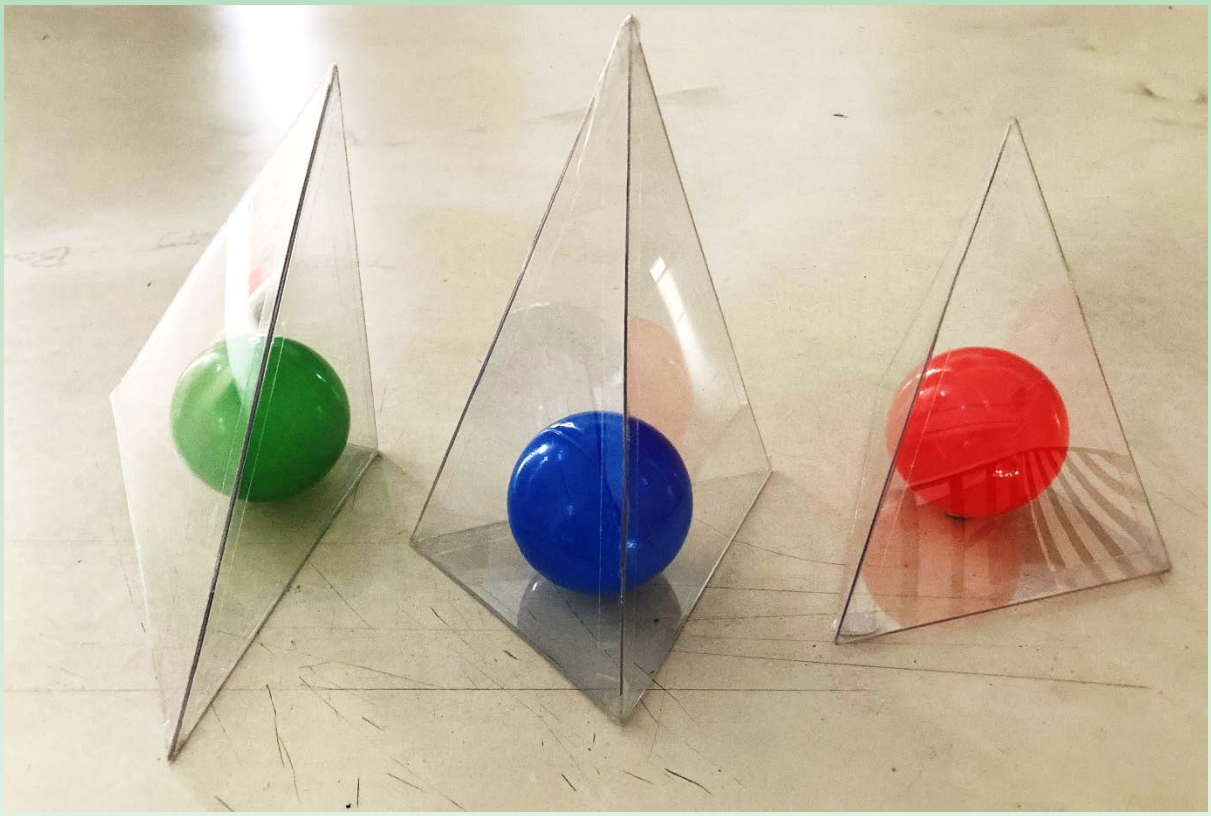
Mathematics Project Competition for Secondary Schools 2023/24



Title: Incenters and Inspheres of Tetrahedrons

Wah Yan College, Kowloon

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1. Introduction

In Form 3, we have learnt the perpendicular bisector property and angle bisector property, and we learnt how to apply these properties to construct the circumcenter and incenter of triangles.

Meanwhile, we know that there is a question in HKDSE 2023 relating to 3D trigonometry, involving tetrahedrons and the circumcenter of triangles.

It inspires us to extend our knowledge on centers of triangles in 2D. How can we construct the circumcenter and incenter of tetrahedrons in 3D?

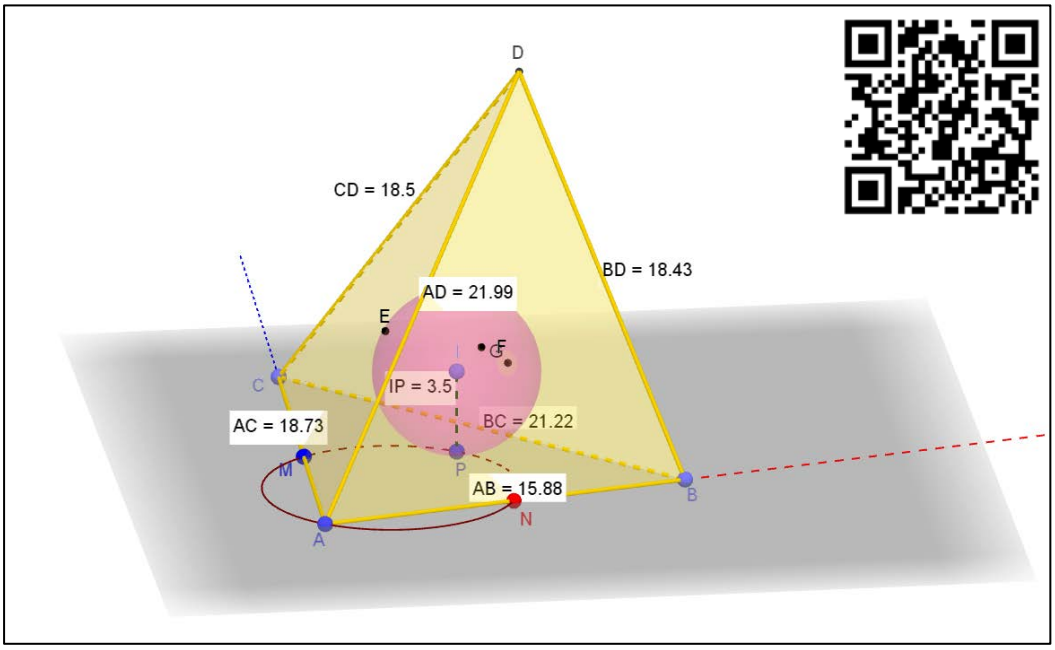
Finally, we find that it is quite easy to construct the circumcenter of tetrahedrons. But the story is totally different for the incenter of tetrahedrons. In this project, we would like to share our finding, and our journey of studying the incenter and insphere of tetrahedrons.

7. Summary

In this project, we show that the incenter of a tetrahedron is very difficult to be located by the angle bisectors between the faces of the tetrahedron. Therefore, we find another way to locate the incenter of the tetrahedron by using 3-D trigonometry instead.

Furthermore, from the work of locating the incenter of the tetrahedron, we discover a way to construct a tetrahedron with given insphere. We use the straightedge and compass construction from 2-D to 3-D and make the construction in GeoGebra 3-D calculator. It let us develop an App to help uses to construct their own tetrahedron with given insphere, with the aid of GeoGebra augmented reality (AR) technology.

GeoGebra App: https://www.geogebra.org/3d/ezaz4g8r



8. References

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