Extract

Foundations of Junior Secondary Pure Geometry

—— (Re)-constructed from Junior Secondary Students' Perspectives

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Abstract

In this project, we have constructed a foundation of junior secondary pure geometry suitable to read for junior students or elementary geometry learners. We have introduced various basic terms, primitive notions, new axioms and definitions that may suit the purpose of proving geometric results learned in Form 1 and 2 throughout this project. Being different from but also inspired by the systems in Euclid's *Elements* and Hilbert's *Foundations of Geometry*, we have strived to keep our work easy to understand intuitively but still sufficiently rigorous to prove the results regarding straight lines, parallel lines and congruent triangles, rather than aiming to lay a foundation with the least number of axioms possible as in these two classics. This project has been arranged in the order of logical dependence, in which only the axioms and results previously mentioned will be required to prove each proposition. Desirable and possible follow-up investigations have also been suggested at the end of this project.

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O. Introduction

Motivation

Geometry is one of the oldest and most important branches of mathematics. We, a group of six junior secondary students, are passionate about the world of geometry. In mathematics lessons, we have been trained to perform deductive proofs. However, after diving deeper into these different geometrical theorems, we have found out that textbooks often lack proofs for these theorems. To remedy this, we decided to seek answers from Euclid's *Elements*.

As Isaac Newton once said, "*It is the glory of geometry that from so few principles, fetched from without, it is able to accomplish so much.*" Euclid's *Elements* is a remarkable work of logical deduction that lays the foundations of geometry and remains a fundamental text in mathematics to this day. Its influence has been far-reaching, shaping the development of mathematics and inspiring generations of mathematicians.

However, Euclid's *Elements* also receives some criticism. James J. Sylvester, a well-known algebraist, once said, "*The early study of Euclid made me a hater of geometry*". It is often too challenging for geometry lovers, particularly junior form students, to seek the roots of geometric knowledge via Euclid's *Elements*.

Thus, our goal is to write a foundation for junior secondary pure geometry for junior secondary geometry lovers. We hope that this can become a reference that would be valued by teachers and fellow students who would like to go on the same pursuit of pure geometry as us. In this project, we will create our own axioms and definitions to (re-)construct a foundation for results pertaining to straight lines, parallel lines and congruent triangles learned in junior secondary geometry curriculum.

Features and Outline

In Section I, we will include theorems related to properties of straight lines. We will introduce the definition of angles, alongside their addition and subtraction. In Section II, we will introduce the concept of parallel and dive further into the theorems proved by the parallel postulate. Lastly, in Section III, we will introduce definitions of congruent triangles and explain how they can be proved and applied. The proposition order mainly follows the current textbooks, thus largely different from the organization in Euclid's *Elements*.

Some of our constructions or transformations are enabled by Hilbert's *Foundation of Geometry*, rather than using circles as in *Elements*. Our notions are purely geometrical, e.g. measures of angles and line segments are not introduced. We will maintain a pure geometrical system in our work.

There are four primitive notions in our project: point, line and congruence of line segments and angles.

IV. Conclusion and Closing Remarks

In this project, we have successfully created a rigorous deductive proof of theorems related to properties of straight lines, parallel lines and congruent triangles with an axiomatic approach. The proofs used in this project are similar to the common approach of junior secondary students, which can be taken as a reference for teachers and fellow students.

Despite this, there are also some limitations and difficulties while conducting the research. Here are some suggestions for further research:

1. Some axioms, for example Axiom III.6 (Axiom of Congruent Triangles), can be simplified and restated to allow a better understanding of the readers.
2. Some axioms, e.g. Axiom of Betweenness, Axiom or Containment established in Hilbert's Foundations of Geometry, can be incorporated to make the system more rigorous.
3. Further research on similar triangles can be done. We desired to include that part in our project, but the definition of proportion may be out of the scope of pure geometry. A better system can be incorporated.
4. Further research on angle sum of polygon can be done. The proofs that we can think of either require the use of circles and/or mathematical induction, which is beyond the scope of junior secondary mathematics curriculum.

V. References

1. Euclid's *Elements* - Clark University:

   http://aleph0.clarku.edu/~djoyce/elements/elements.html

2. *Foundations of Geometry*, by D. Hilbert - Berkeley Math:

   https://math.berkeley.edu/~wodzicki/160/Hilbert.pdf

The project aims at constructing a foundation of junior secondary pure geometry suitable to read for junior students or elementary geometry learners. Being different from but also inspired by the systems in Euclid's Elements and Hilbert's Foundations of Geometry, the team have strived to keep our work easy to understand intuitively but still sufficiently rigorous to prove the results regarding straight lines, parallel lines and congruent triangles, rather than aiming to lay a foundation with the least number of axioms possible as in these two classics.