

MATHEMATICS PROJECT COMPETITION FOR  
SECONDARY SCHOOLS(2021/22)  
CATEGORY A(JUNIOR SECONDARY PROJECT)

## The COVID-19 Testing Problem: Efficiency of Conducting Tests

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# **1 Introduction**

## **1.1 Introduction**

During the pandemic of COVID-19(SARS-CoV-2), the demand for virus testing dramatically increased, and so did the cost and waste caused by the tests. Traditionally, the subjects are tested individually, meaning that the number of tests conducted is the same as the number of people to receive testing. When a massive population needs to be tested, overloading in laboratories will be led and a tremendous amount of waste will be generated. In this paper, we will investigate how to increase the efficiency of conducting tests by pooling the samples together and doing multiple tests. We want to find a better solution for different conditions.

## **1.2 Problem Restatement**

1. Develop mathematical models for pooled testing and use them to minimize the total number of tests required in COVID-19 testing.
2. Determine the maximum infection rate that pooling should be used.
3. Discuss any changes to the model when we consider both grouping and re-groupings of samples, determine the optimum group sizes and the number of re-groupings that minimize the total number of tests required.
4. Compare different grouping's effectiveness by the results.

## **7 Conclusion**

Considering all the strengths and limitations mentioned above, including the efficiency and whether it is easy to obtain the data needed and valuable, we conclude that the Multiple Groupings model method is the most efficient way to conduct tests. In this paper, we successfully obtained a general solution. We developed a computer program for calculating the optimum group sizes of groupings with a given

infection rate and finding the total number of tests required. With this model, we can save many resources and increase efficiency. It makes testing a massive population in a short period possible. We recommend using this method when a test is needed in a huge population.

## **Situation in Hong Kong**

As of today (12/5/2022), according to the Census and Statistics Department of Hong Kong, the total population of Hong Kong is estimated to be about 7.403 million. The infected number of people in the last 14 days provided by the Hong Kong Government is 4184 [10]. As the virus can still spread quickly and some hidden cases may not be found, the infection rate will be higher than expected. We assume that the infected population to be 6000 people. Using our model, with an infection rate of 0.000810, 6 groupings are needed and the total number of tests required is approximately 116184. Since the daily limit of testing in Hong Kong is about 200000 tests, based on our models, it may be a suitable time for conducting Compulsory Universal Testing (CUT). However, due to the limitations that we have mentioned, there may be a discrepancy between the models and reality. We suggest that CUT should be done after the trend goes downwards to 2 digits per day to increase the speed of Hong Kong to resume normal life. Given the large scale of the scheme which calls for precise execution, the Government has to plan the details carefully to ensure sufficient ancillary measures and manpower in place.

## **8 Reflection**

### **Student 1**

Participating in this project is a fun and meaningful experience. When we first started to work on this project, we had no idea about it. Starting from zero, we not only had to face mathematical problems, but we also had to learn all sorts of things like computer programming languages and document preparation software. This project is undoubtedly challenging for us. We are facing many problems when we are working on this project. Fortunately, through our teamwork, we solved all of them. I learned a lot from this project. Overall, I enjoyed participating in this project.

### **Student 2**

Through this project, I learned many new things about mathematics. I never realized that using mathematics could solve such problems in our daily lives. In no way do I disagree with the last great works by the mathematician. At the same time, our team had done something that had similar patterns. I feel honoured and lucky to participate in this project. Finally, I hope I can have the chance to take part in these kinds of projects again in the upcoming future.

### **Student 3**

In this project, I learned about many different skills of Mathematics which is new for me. I never thought that maths could solve problems in our daily life before the project. After participating in this project, I changed my thoughts on Mathematics. Not only can it solve the questions in the book, but it can also solve our daily life problems. Although there are still many things that I don't understand, I will continue to work hard. Finally, I am very grateful to the teacher for giving me this opportunity to try out something that I have not been exposed to.

### **Student 4**

Before joining this project, I couldn't imagine that there were other ways to increase the efficiency of conducting tests. I thought Maths could only be used in calculating the questions in textbooks and it seemed useless to learn complicated Maths. But through this project, I have learned that we can apply the maths we learned in our daily lives and help us to solve our problems. The use of Maths depends on our way of thinking. It broadens my horizons.

### **Student 5**

Since 2019, COVID-19 has spread widely around the world. Therefore, COVID-19 testing is highly required. The goal of our project is to detect the most cases with the least amount of reagents. Before this project, I only knew that by using math, we could solve a lot of problems, but I didn't expect that it can be applied to COVID-19 testing. Some knowledge we have learned in our textbooks can be applied to this epidemic. This report broadened my horizons. If there is another chance, I will participate again next time.

### **Student 6**

It is my pleasure to participate in this project. Our project is based on the current seriousness of the new coronavirus disease, hoping to find a faster and more convenient method for sample testing. When

we first started to work on this project, we had no idea at all, but after several meetings and discussions with my group mates, we gradually developed a direction for this topic. I am very grateful for my group member's support. Without them, I may not know what to do in this project! I learned more about maths like modifying general solutions and calculating the range of probabilities that we didn't learn at school. This is an interesting experience for me.

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