Investigation on Lucky Fractions

By Zoe Cheng, Alicia Cheung, Michelle Ching, Alvina Tse, Vivienne Tsui, Isabella Wan

Diocesan Girls' School

1. Introduction

While we were browsing for fun mathematics problems on the Internet, a question from the Inter-school Mathematics Contest 2013 (Hong Kong Joint School Mathematics Society, 2013), caught our eye.

"While Kevin was doing his Mathematics homework, he encountered a proper fraction, where both denominator and numerator are 2-digit number. He misunderstood the definition of simplifying a fraction: cancelling the same non-zero digit of both denominator and numerator.

Example:

$$\frac{18}{84} \to \frac{1\cancel{8}}{\cancel{8}4} \to \frac{1}{4}$$

However, the simplified fraction (the wrong one) he got has the same value as the correct fraction. Give 2 possible original proper fractions that Kevin encountered."

This aroused our interest in this type of fraction. After some research, we came across the term "anomalous cancellation". Some would call them "lucky fractions". In this project, we will investigate when anomalous cancellation happens in 2-digit numbers and why it is limited to specific numbers. Afterwards, we will extend our findings to 3-digit lucky fractions, finding and examining some interesting properties of these numbers.

8. Conclusion

Investigation was carried out on the properties of 2-digit and 3-digit lucky fractions. The 2-digit cases refers to the equality $\frac{10x+y}{10y+z}=\frac{x}{z}$, where *x*, *y*, and *z* are all non-zero distinct denary digits and $0<\frac{10x+y}{10y+z}<1$. It is proven that *y* has to be either 6 or 9, with only 4 possible cases, namely $\frac{16}{64}$, $\frac{26}{65}$, $\frac{49}{98}$ and $\frac{19}{95}$.

On the other hand, we consider a particular 3-digit cases in which the tens digits of the numerator and the denominator are the same. In other words, we refer to the equality $\frac{100p+10x+q}{100r+10x+s}=\frac{10p+q}{10r+s}$, where *p*, *q*, *r* and *s* are denary digits, $p,\ r\neq 0$, while $0<\frac{100p+10x+q}{100r+10x+s}<1$. It is found that $x=p+q=r+s$ in all bases. Some other interesting properties were also observed in other 3-digit cases with base 10.

In this project, only 2-digit cases and 3-digit cases with the tenths digit cancelled were investigated, while there can be more cases with the unit digit cancelled, or maybe with 4 digits or more. However, due to time restrictions, we were only able to dive into the few cases of anomalous cancellation, but we look forward to extending our exploration in this area of “lucky fractions”.

9. Reflection

This project has deepened our understanding in anomalous cancellation. We were able to find the conditions of anomalous cancellation to occur, through investigating different properties of cases with different bases. When we first came across this fascinating topic through the interesting mathematics problem on the Internet, we were already captivated by the amazing coincidence that helps us to simply delete digits to simplify a fraction. We have learnt simplifying fractions for a long while at school, and anomalous cancellation is an amusing activity that we have never touched upon before that does the same effect for ordinary cancellation. Therefore, we decided on this topic, aiming to find the pattern for this cancellation method to be used.

We were able to find the properties of these special 2-digit fractions at first with ease, with our knowledge of algebra and number theory. We tried using divisibility, substitution, and many different ways we can think of to accurately solve the equations. However, finding all of these specific fractions by hand would be very time consuming. Therefore, we figured out writing a program using C++ to find out the fractions that anomalous cancellation can be applied. Since we used a computer program, it took less time to find the fractions. We surely did experience how difficult it was to formulate equations and write a program for it, but we enjoyed every single minute of it.

This project has aroused all of our groupmates' interest in this topic. With our limited time and resources for this project, we were only able to find cases where $\frac{100p+10x+q}{100r+10x+s}=\frac{10p+q}{10r+s}$. However, we're still currently trying to further investigate similar cases where $\frac{p,q,r}{s,t,r}=\frac{p,q}{s,t}$ or $\frac{p,q,r}{p,s,t}=\frac{q,r}{s,t}$. This project has helped us gain a more conceptual understanding through proof and calculation which might not be obtained through given mathematical formulas, allowing us to look deeper and further into this topic.

All in all, this project is very meaningful and eye-opening. Through this project, we were able to learn to look for different independent ways to prove a result, and most importantly, to cooperate with groupmates to search for answers together through communication. We understand that we have only looked into a small part of this topic, and we will continue to extend the knowledge learnt through this project and challenge ourselves to explore the beauty of mathematics and broaden our horizons.

10. Reference

1. Boas, R. (1972). Anomalous Cancellation. The Two-year College Mathematics Journal, 3(2), 21-24.

2. Hong Kong Joint School Mathematics Society. (2013). Inter-School Mathematics Contest 2013 – Individual Event (Junior Section). Hong Kong Joint School Mathematics Society.

3. Silverman, J. (2013). A friendly introduction to number theory (4th ed.). Boston: Pearson.