Extract

Wong Shiu Chi Secondary School

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1. Introduction

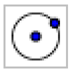
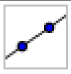
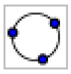
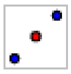
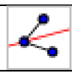
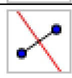
A *compass-and-straightedge construction* is a geometric construction using an idealized compass[1] and an idealized ruler[2] (straightedge) only[3]. The ancient Greek mathematicians discovered methods to do a lot of constructions including perpendicular lines, parallel lines, angle bisectors, etc. However, they also found that some constructions are extremely difficult.

For more than 2000 years, mathematicians struggled to solve three compass-and-straightedge construction problems, namely ‘squaring the circle’, ‘doubling the cube’ and ‘angle trisection’ [1]. Their attempts to solve the problems inspired the development of many branches of mathematics. For example, attempts to solve the problem of doubling the cube have led to the studies of conic sections [2]. These three constructions are nowadays proved impossible.

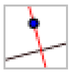
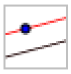
All over the world, students learn compass-and-straightedge constructions at schools. Studying compass-and-straightedge construction helps students to understand geometry and deductive logic. On top of these, students can develop an interest in mathematics and problem-solving. It is quite a shame that the role of this topic is diminishing in the modern curriculum.

First released in 2001, GeoGebra is an interactive mathematics software for learning and teaching geometry. Users can use GeoGebra to perform compass-and-straightedge constructions. It also provides other tools like midpoint, parallel line, angle bisector, etc.

GeoGebra tools used in the project are as follows:

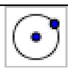
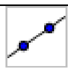
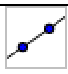
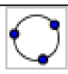
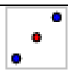
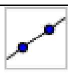
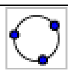
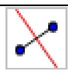
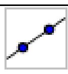
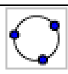
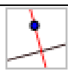
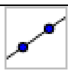
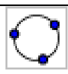
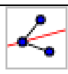
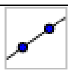
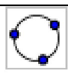
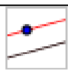
Name	Symbol	Function	GeoGebra icon
Compass	C	constructs a circle with the first selected point as the centre that passes through the second selected point	
Straightedge	S	constructs a straight line that extends endlessly through two selected points	
3-point circle	3	constructs a circle passing through three selected points	
Midpoint	M	constructs the midpoint of two selected points.	
Angle-bisector	A	constructs the bisector of a selected angle.	
Perpendicular bisector	Pb	constructs the perpendicular bisector of the line segment joining two selected points.	

[1] An idealized compass will collapse when lifted from the page. It may not be directly used to copy distances.
[2] An idealized ruler, or straightedge, is infinite in length and has no markings on it.
[3] By the compass equivalence theorem [8], the compass can be replaced by a compass that will not collapse when lifted from the page.

Name	Symbol	Function	GeoGebra icon
Perpendicular line	Pe	constructs the line that is perpendicular to a selected line and passes through a selected point.	
Parallel line	Pa	constructs the line that is parallel to a selected line and passes through a selected point.	

In this project, we will study figures constructible with some combinations of GeoGebra tools. We call these combinations of tools GeoGebra toolsets. We will prove that some toolsets are equivalent to the compass-and-straightedge toolset in a sense that all figures constructible with one toolset are also constructible with the other.

GeoGebra toolsets studied in this project are:

Toolsets	Symbol	GeoGebra icon
Compass and straightedge	CS	 and 
Straightedge, 3-point circle and midpoint	S3M	,  and 
Straightedge, 3-point circle and perpendicular bisector	S3Pb	,  and 
Straightedge, 3-point circle and perpendicular line	S3Pe	,  and 
Straightedge, 3-point circle and angle bisector	S3A	,  and 
Straightedge, 3-point circle and parallel line	S3Pa	,  and 

4. Conclusion: equivalence of toolsets

We can use compass and straightedge to construct the midpoint of two points, the perpendicular bisector of a line segment, the line through a given point and perpendicular to a given line, the bisector of an angle and the line through a given point and parallel to a given line. So, we can use compass and straightedge to construct all figures constructible with any one of the S3M, S3Pb, S3Pe, S3A and S3Pa toolsets (see Figure 36).

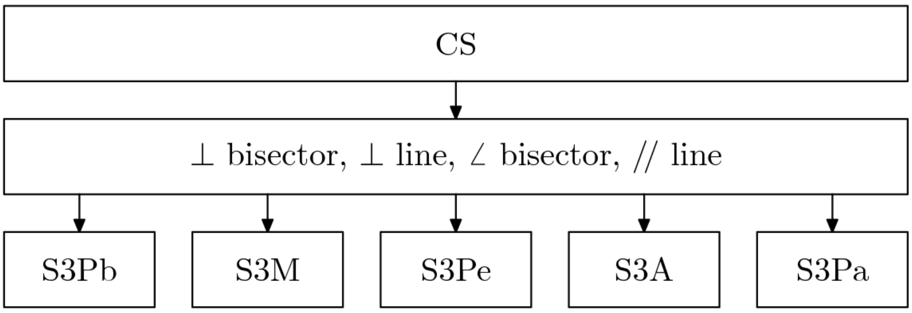


Figure 36

In Chapter 2, we give a method to construct the nine-point circle of an arbitrary triangle using the S3M toolset. Based on the construction of the nine-point circles, we develop methods to construct the orthocentre, the circumcentre and the incentre of the triangle.

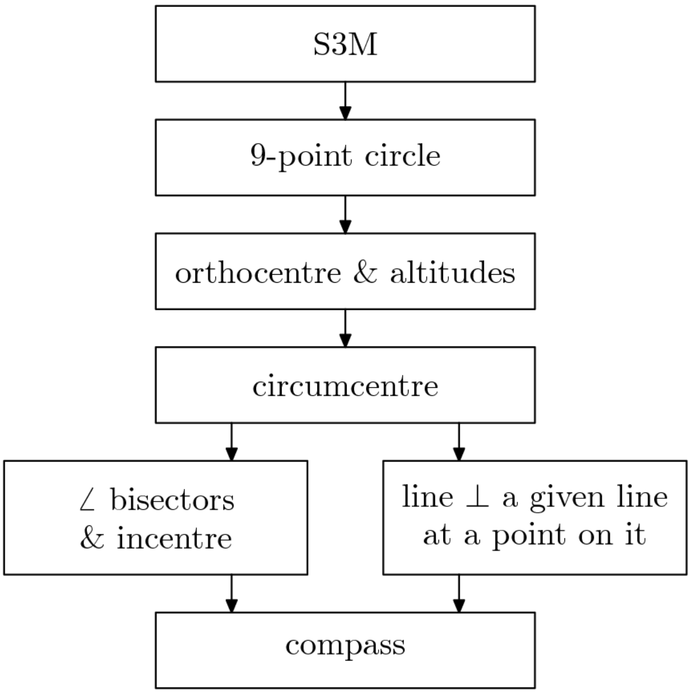


Figure 37

These construction methods help us to construct the line perpendicular to a given line at a given point on it. This enables us to use the S3M toolset to construct any figure that can be constructed with a compass (see Figure 37).

In Chapter 3, we give the methods of constructing the midpoint of two arbitrary points using the S3Pb, S3Pe, S3A and S3Pa toolsets respectively. It means that all figures constructible with the S3M toolsets are constructible with any one of these four toolsets. Therefore, these four toolsets can be used to construct anything that can be constructed with the compass-and-straightedge toolset (see Figure 38).

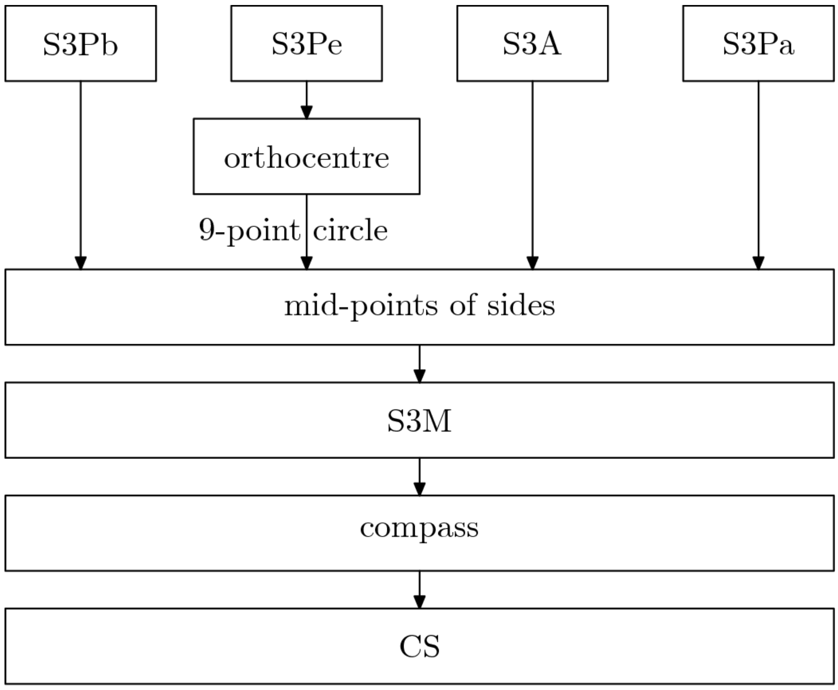


Figure 38

Figure 39 shows the relationship among the six toolsets.

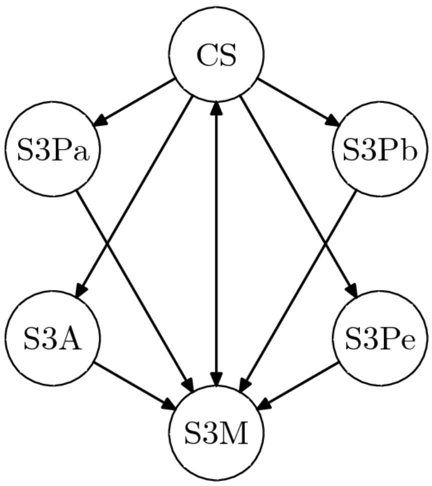


Figure 39

It is now sufficient to conclude that all these six toolsets are equivalent to each other (see Figure 40).

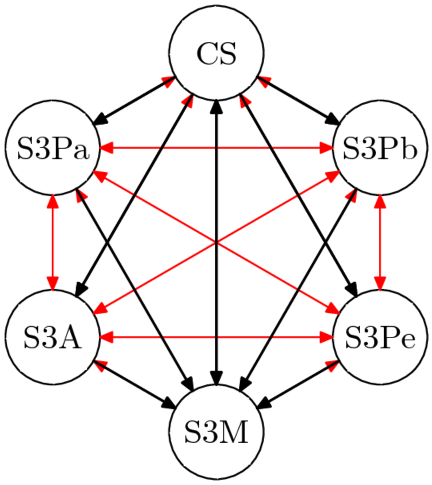


Figure 40

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