Extract

INVESTIGATION AND ANALYSIS OF THE CONDITIONS FOR PROVING CONGRUENT QUADRILATERALS

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A. INTRODUCTION

I have learnt how to prove a pair of congruent triangles from my textbook. There are totally 5 methods, namely: S.S.S., S.A.S., A.S.A., A.A.S. and R.H.S..

After learning them, I asked my teacher, 'Are there any mathematical methods to prove that a pair of quadrilaterals are congruent?'

'The textbook does not mention that. However, you can find it out by yourself.' My teacher answered.

I asked my mum afterwards, 'Are there any methods to prove that a pair of quadrilaterals are congruent?'

'Cut the 2 quadrilaterals out and put them together. If they totally overlap with each other, they must be congruent.' she answered.

Although my mum is correct, her method is not a mathematical proof. Therefore, I started this project to solve this problem.

B. CONGRUENT TRIANGLES

1. Definition of Congruence

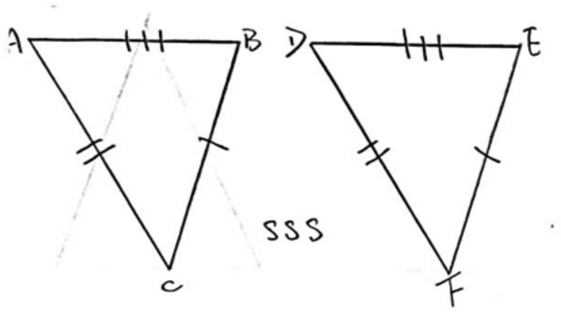
Figures which are of the same shape and size are called congruent figures.

2. How to Prove

To prove a pair of triangles, there are 5 methods, namely S.S.S., S.A.S., A.A.S., A.S.A. and R.H.S..

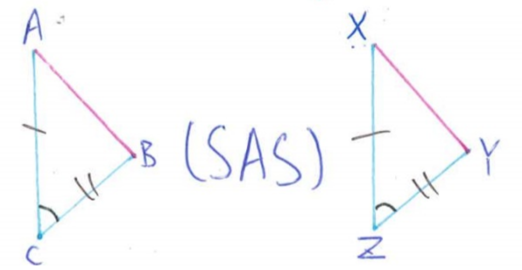
(1) S.S.S. (side-side-side)

$AB = DE$ (given)
$BC = EF$ (given)
$AC = DF$ (given)
Then, $\Delta ABC \cong \Delta DEF$ (S.S.S.)



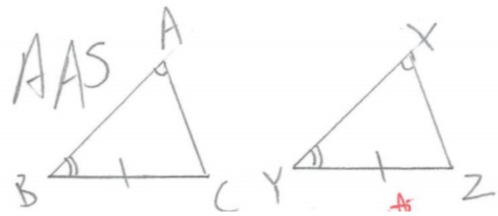
(2) S.A.S. (side-angle-side)

$AC = XZ$ (given)
$BC = YZ$ (given)
$\angle ACB = \angle XZY$ (given)
Then, $\Delta ABC \cong \Delta XYZ$ (S.A.S.)



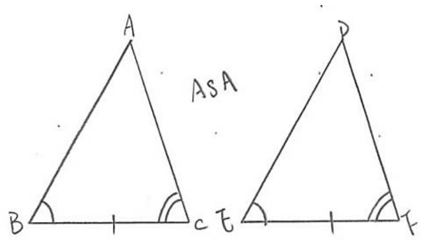
(3) A.A.S. (angle-angle-side)

$\angle BAC = \angle YXZ$ (given)
$\angle ABC = \angle XYZ$ (given)
$BC = YZ$ (given)
Then, $\Delta ABC \cong \Delta XYZ$ (A.A.S.)



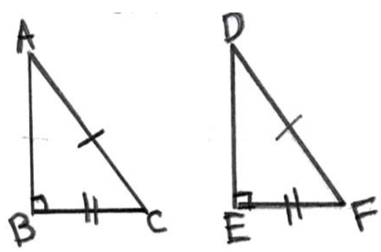
(4) A.S.A. (angle-side-angle)

$\angle ABC = \angle DEF$ (given)
$\angle ACB = \angle DFE$ (given)
$BC = EF$ (given)
Then, $\Delta ABC \cong \Delta DEF$ (A.S.A.)



(5) R.H.S. (right angle-hypotenuse-side)

$\angle ABC = \angle DEF = 90°$(given)
$AC = DF$ (given)
$BC = EF$ (given)
Then, $\Delta ABC \cong \Delta DEF$ (R.H.S.)



C. CONGRUENT QUADRILATERALS

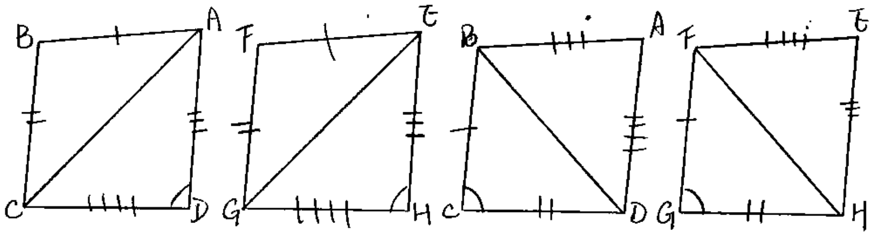
1. Definition of Quadrilaterals

A quadrilateral has 4 angles formed by its 4 sides. The prefix ‘quad-‘ means ‘4’ and ‘lateral’ is derived from Latin word ‘side’. So a quadrilateral is a four-sided polygon.

2. How to Prove

Every quadrilateral can be split into 2 triangles. If the 2 triangles are congruent to another 2 triangles from another quadrilateral, then the 2 quadrilaterals are congruent.

3. Why the Prove Works



$BC = FG$ (given)
$CD = GH$ (given)
$\angle C = \angle G$ (given)
Then, $\Delta BCD \cong \Delta FGH$ (S.A.S.)

$AB = EF$ (given)
$AD = EH$ (given)
$BD = FH$ (corr. sides, $\cong\Delta$s)
Then, $\Delta ABD \cong \Delta EFH$ (S.S.S.)

Therefore, $ABCD \cong EFGH$.

$AD = EH$ (given)
$CD = GH$ (given)
$\angle D = \angle H$ (given)
Then, $\Delta ACD \cong \Delta EGH$ (S.A.S.)

$AB = EF$ (given)
$BC = FG$ (given)
$AC = EG$ (corr. sides, $\cong\Delta$s)
Then, $\Delta ABC \cong \Delta EFG$ (S.S.S.)

Therefore, $ABCD \cong EFGH$.

We can see that no matter how to cut the quadrilateral into 2 triangles, the triangles are all congruent and the quadrilateral will be congruent.

E. CONCLUSION

After our analysis, we found out that if 2 quadrilaterals match the following conditions, the quadrilaterals will be congruent. The conditions are:

1. S.S.S.S.A.
2. S.A.S.A.S.
3. A.S.S.A.A.
4. A.S.A.S.A. (with 2 unparallel sides not given)

Now, I can tell my teacher that although the problem is challenging, I can solved it! This project is fun and interesting. I am willing to take another challenge in the future.

F. REFERENCE

1. K.H.Yeung, C.M. Yeung, Y.F. Kwok, H.Y. Cheung, K.C. Chan, F.C. Tong (2015). *Congruence and Similarity*, Mathematics in Action (3rd edition). Pearson.
2. Lang S., Murrow G. (1988) *Congruent Triangles. In: Geometry*. Springer, New York, N.Y.