Extract

The World of Possibilities in Dice

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Introduction

As the Chinese welcome Lunar New Year with family gatherings and celebrations, family games making use of a fair die have emerged. These simple variations of dice games inspire us to delve deep into the topic of probability surrounding the die. Many of us might be familiar with idioms such as 'a roll of the dice' or 'the die is cast' derived from such games. The modern day cubical dice have actually been dated back as early as 600 b.c.; their long history provides us with much food for thought in applying these concepts into our daily lives. On this account, we begin questioning ourselves whether one can make decisions solely based on the result of rolling a die. How even does rolling a fair die provide us with even probabilities at complete random? And why have people relied on this method for centuries, even millennia?

Below are the everyday life situations with the use of a fair die which we will be investigating.

Question 1: If we have a fair 6-faced die with x books to choose from, what are the methods to choose a book randomly?

Question 2: If we have a fair die with$y$faces and $x$books to choose from, is it still possible to choose a book solely based on the result of rolling the die? Is it applicable for all values of x and y?

Summary

With x books, we came up with two methods to choose one of the books randomly based on the result of rolling a fair die with 6 faces. The two methods make use of divisibility and modular arithmetic respectively. To choose 1 out of x books using a y-faced die, we could use the two aforementioned methods as well. On top of that, we have come up with two more methods. One of them is "changing base", where we change the base of x into either the largest factor of y that is smaller than x or 2. Another method is the "Pile A method", where we roll the die for each book and put the books with result 1 in a pile. These different methods vary in efficiency and simplicity, so we have compared their efficiency by considering the number of rolls and digits. To determine which general algorithm is suitable, we have to consider different sets of x and y. The categories of sets of x and y we have to consider are stated clearly under the method we are using.

Conclusion

Investigation was carried out on the methods of choosing 1 book out of x books using a fair 6-faced die and a fair y-faced die respectively. Since 6 is included in the many values y can take up, the methods we have come up with using a y-faced die can be applied to the former investigation. The 4 methods are namely:

1) Divisibility. For this method, we add re-roll options to x so that x is divisible by y or is a multiple of the factors of y, then divide the number of faces of the die and the re-roll options to each book.

2) Rolling z times, with z satisfying the equation $y^z \equiv 0 (\mathrm{mod}\, x)$. For this method, we roll the die for z times such that the total number of outcomes can be assigned evenly to each book.

3) Changing the base of x, which can be split into 2 methods: (A) taking the largest factor of y that is smaller than x as the base and (B) taking 2 as the base. For this method, we start by rolling the die to determine the number we should write down for the rightmost place value and work our way towards the left.

4) Pile A method. For this method, we roll the die for every book and put the book with result 1 into pile A, repeating the process until there is one book left.

It is found that some methods are only applicable to particular ranges of x and y and each method is efficient in different situations. This has been detailed in the last section of the report.

In this project, we have made use of numerous mathematical concepts including modular arithmetic, numeral systems and sum to infinity of geometric sequences etc. Due to our limited knowledge in the subject, we were unable to find an efficient method for changing the base of x into 2 because we always have to roll for the largest place value to keep the probability of getting any 1 book consistent in all situations. There could be a plethora of methods and ways of improving efficiency that were not mentioned in the report due to time constraints, so we look forward to extending our investigation into the fascinating calculations of probability a die can offer.

Reflection

The topic of probability has always been a part of the mathematics curriculum, yet we never had the opportunity to delve deep into the calculations of the probability a die entails. Whether we are considering the classic 6-faced die or a 50-faced die, the different methods to guarantee a fair result enthrals us, ranging from changing bases and adding re-roll options, to modular arithmetic and the Pile A method we have discovered. Not only did we garner a deeper understanding of probability, we have also touched upon other mathematical concepts that we didn't think would correlate with this investigation, some examples being numeral systems and sum to infinity formula for geometric sequences.

In the beginning of our investigation, we explored the different methods we can use to solve the two core questions raised: using 1) a fair 6-faced die and 2) a fair y-faced die to select 1 out of x books. During the process of calculating different cases, there were countless times of trial and error and intellectually stimulating discussions. We experienced how many complications in calculating probabilities as the number of possible situations is unlimited. We have also compared the efficiency and usages of the methods we introduced, all of which we believe has their own benefits.

To add on, we also relished every moment of the investigating process, from sharing our countless attempts with group mates to work out the solution, to helping each other out in areas we were not familiar with. Amid the coronavirus pandemic, which makes it difficult for the whole group to meet together face-to-face, we have effectively made use of various online communication services such as Zoom and Gmail to exchange ideas. Thankfully, everyone is still able to contribute to the project to complete the project with our best efforts . Due to time constraints and our limited knowledge, we were only able to focus on and come up with the aforementioned methods in our calculations. However, we are more than delighted to explore further into the endless topic of probability and welcome any insights into the topic.

All in all, this project has helped us garner more conceptual understanding on probability and most importantly, provoked our interest in mathematics. Perhaps next time when we play a game of dice, we will be reminded of the scope of imagination this project has furnished us with and the beauty of mathematics.

In the third round, the probability of Book 1 passing and Book *m* failing to pass is $\frac{1}{6}\times\frac{5}{6}$.

Therefore, P (Case 3c)$= [(\frac{1}{6})^2 \times \frac{5}{6} \times 2] \times [(\frac{1}{6})^2 + (\frac{5}{6})^2] \times (\frac{1}{6} \times \frac{5}{6})$

$$= (\frac{5}{216} \times 2) \times \frac{26}{36} \times \frac{5}{36} \times 2$$

$$= \frac{5}{108} \times \frac{26}{36} \times \frac{5}{36}.$$

Therefore, to sum up,

P(choosing a book from 3 in the 3rd round)

$$= \frac{126}{216} \times \frac{126}{216} \times \frac{25}{216} + \frac{126}{216} \times \frac{5}{108} \times \frac{5}{36} + \frac{5}{108} \times \frac{26}{36} \times \frac{5}{36}.$$

Reference List

1. Kenneth H. Rosen (2010). Elementary number theory (6th edition). Pearson.

2. S.M. Ross (2007). Introduction to Probability Models (9th ed.). Academic Press